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## Neutrinos with Zee-Mass Matrix in Vacuum and Matter

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### ABSTRACT

Neutrino mass matrix generated by the Zee (radiative) mechanism has zero (in general, small) diagonal elements and a natural hierarchy of the nondiagonal elements. It can be considered as an alternative (with strong predictive power) to the matrices generated by the see-saw mechanism. The propagation in medium of the neutrinos with the Zee-mass matrix is studied. The flavor neutrino transitions are described analytically. In the physically interesting cases the probabilities of transitions as functions of neutrino energy can be represented as two-neutrino probabilities modulated by the effect of vacuum oscillations related to the small mass splitting. Possible applications of the results to the solar, supernova, atmospheric and relic neutrinos are discussed. A set of the predictions is found which could allow to identify the Zee-mass matrix and therefore the corresponding mechanism of mass generation.

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# 1 Introduction

There are a number of mechanisms of the neutrino mass generation which relate naturally the smallness of the masses to the neutrality of neutrinos. Those include the see-saw mechanism [1],[2] different radiative mechanisms [3], [4], [5], [6], [7], [8], the tree level mass generation by Higgs triplet having small induced VEV [9], mass generation by nonrenormalizable Planck scale interaction [10]. In general several mechanisms contribute to the neutrino masses simultaneously, moreover their contributions can be comparable or one of them dominates. Different mechanisms imply different symmetries and particle contents of the theory.

There are some indications of the existence of tiny neutrino masses and lepton mixing related to the solar, atmospheric and relic neutrinos. The suggested mechanisms can rather easily generate the neutrino masses (mass squared differences) and mixings in some or even all regions of these “positive indications”. What can be learned about the origin of neutrino masses and mixings from the existing, and mainly, from future experiments with solar, atmospheric, supernova neutrinos as well as from the accelerator experiments? Can one conclude something on the mechanism of the neutrino mass generation? In principle, different mechanisms result in different structures of mass matrices and consequently in different features of the neutrino propagation in vacuum and matter. For the identification of the mechanism it is crucial to study the effects of mixing of all three neutrinos. The propagation of three neutrinos with strong hierarchy of masses and mixings typical for simplest versions of the see-saw mechanism has been widely discussed before[11].

In this paper we will study the properties of propagation in medium of the neutrinos  $\nu_f^T \equiv (\nu_e, \nu_\mu, \nu_\tau)$  having the Majorana mass matrix:

$$M = \begin{vmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{vmatrix}, \quad m_{e\mu} \ll m_{\mu\tau}, m_{e\tau}. \quad (1)$$

Its crucial features are: zero (in general small) diagonal elements and strong hierarchy of the nondiagonal elements. Such a matrix appears in a class of models with radiative mass generation. The first and the simplest version which naturally results in structure (1) had been suggested long time ago by Zee [3] and we will call matrix (1) Zee-mass matrix.

The Zee mechanism implies the existence of charged scalar field,  $S$ , singlet of the  $SU(2)$

and two doublets of Higgs bosons  $\Phi_u$ ,  $\Phi_d$ . The interactions responsible for the neutrino mass generation are

$$\sum_{\alpha\beta} f_{\alpha\beta} \Psi_{\alpha L}^T C \tau_2 \Psi_{\beta L} S^\dagger + \sum_{\alpha} \frac{m_{\alpha}}{v_d} \bar{\Psi}_{\alpha L} l_{\alpha R} \Phi_d + M_{ud} \Phi_u \tau_2 \Phi_d S^\dagger + h.c.,$$

( $\alpha, \beta = e, \mu, \tau$ ), where  $\Psi_{\alpha L}$  is the lepton doublet,  $l_{\alpha R}$  is the right handed component of the charge lepton,  $m_{\alpha}$  is its mass, and  $M_{ud}$  is the mass parameter. As a consequence of the gauge symmetry the couplings of the singlet  $S$  to lepton doublets are antisymmetric:  $f_{\alpha\beta} = -f_{\beta\alpha}$ . Only one doublet,  $\Phi_d$ , with vacuum expectation  $v_d$  gives masses to the charged leptons. The indicated interactions generate at one loop level the elements of the mass matrix (1) [3] [12]

$$m_{\alpha\beta} \simeq \frac{f_{\alpha\beta}}{16\pi^2} (m_{\alpha}^2 - m_{\beta}^2) \frac{I}{v}, \quad I \equiv \frac{v_u}{v_d} \cdot \frac{M_{ud} v}{M_2^2 - M_1^2} \cdot \ln \frac{M_2^2}{M_1^2}, \quad (2)$$

where  $M_1$ ,  $M_2$  are the masses of charged scalars,  $v \equiv \sqrt{v_u^2 + v_d^2}$  is the electroweak scale and  $v_u$  is the vacuum expectation of  $\Phi_u$ . From (2) one finds the relations between the masses:

$$\frac{m_{\mu\tau}}{m_{e\tau}} \simeq \frac{f_{\mu\tau}}{f_{e\tau}}, \quad \frac{m_{e\mu}}{m_{e\tau}} \simeq \frac{f_{e\mu}}{f_{e\tau}} \left( \frac{m_{\mu}}{m_{\tau}} \right)^2, \quad (3)$$

i.e. the element  $m_{e\mu}$  turns out to be naturally suppressed in comparison with the two other elements thus reproducing the structure (1), unless  $f_{\alpha\beta}$  have strong inverse flavor hierarchy.

The matrix (1) appears also in some modifications of the Zee-mechanism. Instead of one scalar, three charged scalars,  $S_{e\mu}$ ,  $S_{e\tau}$ ,  $S_{\mu\tau}$ , are introduced in [6] that carry double lepton charges. Moreover, the lepton number can be violated spontaneously by the VEV of new neutral singlet which has the coupling  $\Phi_u \Phi_d S^+ S^0$  [6]. Three boson coupling can be generated in one loop [7], being suppressed at tree level by a discrete symmetry. The general properties of models which result in the matrix (1) are (i) the existence of one (or several) charged scalar fields singlets of  $SU_2$ , (ii) the generation of masses of the charged leptons at tree level by only one Higgs doublet. Last feature explains simultaneously a suppression of flavor changing transitions and can be related to a certain discrete symmetry.

As can be shown, the properties of propagation are practically the same for a more general form of mass matrix with nonzero diagonal elements provided such elements do not exceed appreciably  $m_{e\mu}$ . The appearance of these diagonal elements can be related, e.g. to the violation of the discrete symmetry in the interactions of Higgs scalars. (The restrictions on flavor changing neutral currents admit  $m^{diag} \sim m_{e\mu}$ ). They can result also from the

see-saw contributions or from the radiative effects related to the R-parity breaking. Note that such a matrix as well as the original Zee-matrix have an approximate  $L_e + L_\mu - L_\tau$  lepton number conservation. This allows for a large mixing of e- and  $\mu$ - flavors. The  $\tau$  flavor turns out to be singled out.

The properties of the Zee mass matrix in vacuum have been studied by Wolfenstein [13]. Here we consider the matter effects and confront the results with the existing data. The paper is organized as follows. In sect.2 we summarize the properties of the Zee matrix and the oscillations induced by this matrix in vacuum. In sect.3 the properties of the Zee matrix in matter will be considered. Sect.4 is devoted to the conversion induced by the Zee matrix in matter. The application will be discussed in sect.5. Sect.6 contains the outlook and the discussion.

## 2 Zee mass matrix in vacuum

Instead of  $m_{ij}$  it is convenient to introduce basic mass scale,  $m_0$ ,  $e\mu$ - mixing angle,  $\theta$ , and small parameter,  $\epsilon$ , as<sup>1</sup>:

$$\begin{aligned} m_0 &\equiv \sqrt{m_{e\tau}^2 + m_{\mu\tau}^2}, \quad \sin \theta \equiv \frac{m_{e\tau}}{m_0} = \frac{f_{e\tau}}{\sqrt{f_{e\tau}^2 + f_{\mu\tau}^2}}, \\ \epsilon &\equiv \frac{m_{e\mu}}{m_0} = \left(\frac{m_\mu}{m_\tau}\right)^2 \frac{f_{e\mu}}{\sqrt{f_{e\tau}^2 + f_{\mu\tau}^2}}. \end{aligned} \quad (4)$$

In terms of these parameters the mass matrix (1) can be rewritten as

$$M = m_0 \begin{vmatrix} 0 & \epsilon & \sin \theta \\ \epsilon & 0 & \cos \theta \\ \sin \theta & \cos \theta & 0 \end{vmatrix}. \quad (5)$$

Let us summarize the properties of this matrix. The eigenvalues of (5) equal to [13]

$$m_1 = -m_0 \epsilon \sin 2\theta, \quad m_{3,2} = m_0 \left( \pm 1 - \frac{1}{2} \epsilon \sin 2\theta \right), \quad (6)$$

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<sup>1</sup>In contrast with Wolfenstein [13], we use  $\sin \theta \leftrightarrow \cos \theta$ ,  $\epsilon \rightarrow \delta$ .

and the mixing matrix in vacuum which relates the flavor and the mass eigenstates  $\nu_f = S_0 \nu$  ( $\nu \equiv \nu_1, \nu_2, \nu_3$ ) is

$$S_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}c & s & s \\ -\sqrt{2}s & c & c \\ 0 & 1 & -1 \end{pmatrix} + O(\epsilon), \quad (7)$$

here (and everywhere below)  $s \equiv \sin \theta$ ,  $c \equiv \cos \theta$ .

Since  $\epsilon \ll 1$ , two states,  $\nu_2$  and  $\nu_3$ , are approximately degenerate, and their masses ( $\sim m_0$ ) are much larger than the mass of  $\nu_1$ . Moreover  $\nu_2$  and  $\nu_3$  have opposite CP-parities thus forming the pseudo Dirac neutrino. Their mass squared difference is

$$\Delta m_{23}^2 = 2\epsilon \sin 2\theta m_0^2 \ll m_0^2. \quad (8)$$

Mixing of the electron and the muon neutrinos is determined by  $\theta$ , it becomes maximal at  $m_{e\tau} \simeq m_{\mu\tau}$  ( $f_{e\tau} = f_{\mu\tau}$ ). The tau neutrino is mixed almost maximally with the combination  $\nu_2^0 = \sin \theta \nu_e + \cos \theta \nu_\mu$ ; the deviation from maximal mixing is proportional to  $\epsilon^2$ :

$$1 - \sin^2 2\theta_{\tau 2} = \frac{1}{4}\epsilon^2 \sin 2\theta. \quad (9)$$

The mass squared differences for  $\nu_1$  component equal  $\Delta m_{12}^2 \approx \Delta m_{13}^2 \approx m_0^2$  and the ratio of mass differences (see (6)) is determined by  $\epsilon$ :

$$\frac{\Delta m_{23}^2}{\Delta m_{12}^2} = 2\epsilon \sin 2\theta. \quad (10)$$

Consequently, the Zee-matrix has two crucial features: it gives naturally two different scales for the mass squared differences and practically maximal mixing between the two heaviest components.

In vacuum the propagation of neutrinos having the Zee-mass matrix results in superposition of oscillations with two different oscillation lengths; the ratio of lengths is determined by (10) [13]. For small distances,  $L \ll 4\pi E / \Delta m_{23}^2$ , the task is reduced to two neutrino oscillations  $\nu_e \leftrightarrow \nu_\mu$  with depth  $\sin^2 2\theta$ . The probabilities of  $3\nu$ -oscillations can be immediately found from (6) (7). For example, the averaged survival probabilities  $P(\nu_e \rightarrow \nu_e)$  and  $P(\nu_\mu \rightarrow \nu_\mu)$  are [13]

$$P(\nu_e \rightarrow \nu_e) = c^4 + \frac{1}{2}s^4, \quad P(\nu_\mu \rightarrow \nu_\mu) = s^4 + \frac{1}{2}c^4. \quad (11)$$

Note that the effect of the third neutrino is reduced to the factor of  $1/2$  in (11) which comes from the lost of the coherence in the maximally mixed neutrino state.

### 3 Properties of Zee mass matrix in matter: levels and mixing

*Evolution equation. Effective Hamiltonian.* In matter the evolution equation of the neutrinos  $\nu_f^T \equiv (\nu_e, \nu_\mu, \nu_\tau)$  can be written as [14]

$$i\frac{d\nu_f}{dt} = H\nu_f, \quad H \simeq \frac{M^2}{2E} + H_{matter}, \quad (12)$$

where  $H$  is the effective Hamiltonian for ultrarelativistic neutrino,  $M^2$  is the vacuum mass matrix (5) squared,  $E$  is the energy of neutrino and  $H_{matter}$  is the matrix describing matter effect:  $H_{matter} = \text{diag}[\sqrt{2}G_F n_e, 0, 0]$ . Here  $G_F$  is the Fermi constant and  $n_e$  is the concentration of electrons. In (12) we have neglected the high order electroweak effects which lead also to the splitting of  $\nu_\mu - \nu_\tau$  levels [17] and suggest that the concentration of neutrinos in medium is small (so that neutrino - neutrino scattering which in particular, generates the nondiagonal elements of  $H_{matter}$ [18] can be neglected). Substituting the explicit expressions for  $M^2$  and  $H_{matter}$  in (12) one gets

$$H = \frac{m_0^2}{2E} \begin{vmatrix} s^2 + \epsilon^2 + \rho & sc & \epsilon c \\ sc & c^2 + \epsilon^2 & \epsilon s \\ \epsilon c & \epsilon s & 1 \end{vmatrix}, \quad \rho \equiv \sqrt{2}G_F n_e \frac{2E}{m_0^2}. \quad (13)$$

Diagonal elements of  $H$  determine the flavor energy levels, and as the density changes, there are two crossings of flavor levels. The resonance (crossing) densities are:

$$\rho_{e\mu} = \cos 2\theta, \quad \rho_{e\tau} = \cos^2 \theta, \quad (14)$$

for  $\nu_e - \nu_\mu$  and  $\nu_e - \nu_\tau$  levels correspondently (see fig.1). Evidently  $\rho_{e\tau} > \rho_{e\mu}$ ; the distance between crossing points ( $\rho_{e\tau} - \rho_{e\mu} = s^2$ ) is always smaller than the width of the  $e - \mu$  resonance ( $\Delta\rho \sim \tan 2\theta$ ). Consequently, there is a strong influence of the resonance related to the largest  $e - \mu$ -mixing on that stipulated by  $e - \tau$ -mixing (fig. 1).

*Mixing matrix in matter, level crossing scheme.* Let us introduce the neutrino eigenstates in matter,  $\nu_{im}$ , and corresponding eigenvalues  $H_i$ , ( $i = 1, 2, 3$ ) as the eigenstates and the eigenvalues of  $H$ . The eigenstates and the flavor states are related by mixing matrix in

matter,  $S(\rho)$ :  $\nu_f = S(\rho)\nu_m$ . The matrix  $S(\rho)$  and the eigenvalues  $H_i$  are determined by the diagonalization condition  $S(\rho)^\dagger H(\rho)S(\rho) = \text{diag}\{H_1, H_2, H_3\}$ .

Let us find  $S(\rho)$  and  $H(\rho)$ . As follows from (13) in zero order over  $\epsilon$  ( $m_{e\mu} = 0$ ) the state  $\nu_\tau$  decouples and the task is reduced to two neutrino case. The Hamiltonian  $H^0 \equiv H(\epsilon = 0)$  is diagonalized by the rotation

$$S_{12}(\theta_m) = \begin{vmatrix} \cos \theta_m & \sin \theta_m & 0 \\ -\sin \theta_m & \cos \theta_m & 0 \\ 0 & 0 & 1 \end{vmatrix}, \quad (15)$$

where  $\theta_m$  is the  $e\mu$  - mixing angle in matter fixed by:

$$\tan 2\theta_m = \frac{\sin 2\theta}{\cos 2\theta - \rho}. \quad (16)$$

The eigenvalues of  $H^0$  which correspond to the rotated states  $\nu_m^0 \equiv (\nu_{1m}^0, \nu_{2m}^0, \nu_\tau)$  can be written in the units  $\frac{m_0^2}{2E}$  as

$$H_{2(1)}^0 = \frac{1}{2} \left[ 1 + \rho \pm \sqrt{(1 + \rho)^2 - 4c^2\rho} \right], \quad H_\tau = 1. \quad (17)$$

The level splitting,  $\Delta H_{12} \equiv H_2^0 - H_1^0 = \sqrt{(\cos 2\theta - \rho)^2 + \sin^2 2\theta}$ , is minimal in resonance (14):  $\Delta H_{12}(\rho_{e\mu}) = \sin 2\theta$ .

At  $\rho = 0$ , one gets from (17):  $H_1^0 = 0$ , and  $H_2^0 = H_\tau = 1$ , i.e. the levels  $H_2^0$  and  $H_\tau$  cross at zero density (fig.1). Due to the strong influence of  $e\mu$ -mixing on  $e\tau$ -mixing the resonance for  $\nu_\tau$  is shifted from  $\rho_{e\tau}$  to  $\rho = 0$ .

The elements of the Hamiltonian (13) proportional to  $\epsilon$  give the corrections to the above level scheme which become important when  $\rho \rightarrow 0$ , in particular they induce the mixing of  $\nu_\tau - \nu_{2m}^0$  states. Using (13) and the mixing matrix (15) one finds the Hamiltonian in the basis of the states  $\nu_m^0 \equiv (\nu_{1m}^0, \nu_{2m}^0, \nu_\tau)$ . Performing then an additional rotation,  $S_{13}(\alpha)$ , in  $\nu_{1m}^0, \nu_\tau$  space, on the angle  $\alpha$  determined by

$$\tan 2\alpha = \frac{2\epsilon \cos(\theta + \theta_m)}{1 - H_1^0 - \epsilon^2}, \quad (18)$$

$(\nu_m^0 \rightarrow \nu'_m, \nu_m^0 = S_{13}(\alpha)\nu'_m)$ , and  $\nu'_m \equiv (\nu_{1m}, \nu_{2m}^0, \nu'_\tau)$ ) one gets the Hamiltonian which de-

scribes the conversion at small densities

$$H \simeq \begin{vmatrix} H_1^0 - \delta & 0 & 0 \\ 0 & H_2^0 & \epsilon \sin(\theta + \theta_m) \cos \alpha \\ 0 & \epsilon \sin(\theta + \theta_m) \cos \alpha & 1 + \epsilon^2 + \delta \end{vmatrix} + O(\epsilon^3). \quad (19)$$

Here

$$\delta \equiv -(1 - H_1^0) \sin^2 \alpha + \epsilon \cos(\theta + \theta_m) \sin 2\alpha$$

is of the order of  $\epsilon^2$ , the values of  $\theta_m$ ,  $H_1^0$ ,  $H_2^0$  are determined in (16 - 17).

According to (18) the angle  $\alpha$  being also of the order of  $\epsilon$  increases with density:

$$\tan 2\alpha = \begin{cases} 2\epsilon \cos 2\theta, & \rho = 0 \\ 2\epsilon(c + s)^{-1}, & \rho = \cos 2\theta \\ 2\epsilon/s, & \rho \gg 1 \end{cases}$$

There is no resonant enhancement of  $\alpha$ .

The Hamiltonian (19) is diagonalized by the rotation  $S_{23}(\beta)$  in the  $(\nu_{2m}^0, \nu'_\tau)$ -space by the angle  $\beta$  determined from

$$\tan 2\beta \approx \frac{2\epsilon \sin(\theta + \theta_m) \cos \alpha}{1 - H_2^0 + \delta} \approx \frac{2\epsilon \sin 2\theta}{-\epsilon^2 \sin^2 2\theta - \rho s^2} \approx \frac{4\epsilon}{\rho \tan \theta}. \quad (20)$$

Here we have taken into account that in the denominator  $\delta$  can be neglected everywhere except small region around  $\rho \approx 0$  and that  $H_2^0 \approx 1 + \rho s^2$  for  $\epsilon^2 \ll \rho \ll 1$  as follows from (17). Obviously,  $\tan 2\beta \ll 1$  for  $\rho \gg 4\epsilon / \tan \theta$ . If  $\rho \gg 1$  one has  $H_2^0 \approx \rho$  and the angle  $\beta$  is even more strongly suppressed  $\tan 2\beta \approx 2\epsilon/\rho$ . At  $\rho = 0$  second expression in (20) reproduces the result (9).

The total mixing matrix in matter that diagonalizes the original Hamiltonian (13) is  $S = S_{12}(\theta_m) \cdot S_{13}(\alpha) \cdot S_{23}(\beta)$ . The eigenstates  $H_2$  and  $H_3$  can be found as the eigenstates of  $2 \times 2$  submatrix of (19),  $H_1 = H_1^0 - \delta$ . Two angles,  $\theta_m$  and  $\beta$ , undergo the resonant enhancement in different density regions.

*Two density regions.* It is possible to divide the whole density region into two parts so that in each part the three level mixing is reduced to two level mixing. Indeed, let us define the density

$$\rho_b \equiv \frac{4\epsilon}{\tan \theta} \quad (21)$$

which fixes the width of the  $\nu_\tau - \nu_{2m}^0$  - resonant layer ( $\Delta\rho_R = 2\rho_b$ ) (for  $\rho < \rho_b$  one has  $\sin^2 2\beta < 1/2$ ). If  $\rho_b \ll 1$ , the regions of small and large densities can be introduced.

(i). Large density region:  $\rho \gg \rho_b$ . Here  $\alpha, \beta \sim \epsilon$  and in the lowest approximation the mixing matrix is  $S \approx S_{12}(\theta_m)$ . The state  $\nu_\tau$  decouples; the dynamics of propagation is determined by change of  $\theta_m$ .

(ii). Small density region:  $\rho \lesssim \rho_b$ . The change of mixing is determined by the angle  $\beta$ , whereas two other angles vary weakly (even if  $\theta = 45^\circ$ ) coinciding practically with vacuum values:  $\theta_m \approx \theta$ ,  $\tan 2\alpha \approx 2\epsilon \cos 2\theta$ . Consequently, in the first approximation one has:  $S = S_{12}(\theta) \cdot S_{23}(\beta)$ ;  $\nu_{1m}^0$  decouples and the dynamics of level crossing is determined by  $2 \times 2$  submatrix of (19).

For extremely large densities,  $\rho \gg 1$ , :  $\theta_m \approx \pi/2$ ,  $\beta \approx 0$  and  $\alpha \approx \epsilon/s$  so that

$$S_m(\rho \gg 1) \simeq \begin{vmatrix} 0 & 1 & 0 \\ -1 & 0 & \frac{\epsilon}{s} \\ \frac{\epsilon}{s} & 0 & 1 \end{vmatrix}, \quad (22)$$

i.e.  $\nu_e$  state decouples being composed of the eigenstate  $\nu_{2m}$ , whereas  $\nu_\mu$  and  $\nu_\tau$  are mixed with the angle  $\alpha \sim \epsilon/s$ .

*Other possibilities.* In special case of equal  $f_{\alpha\beta}$ , one has  $m_{e\tau} = m_{\mu\tau}$ , and therefore,  $s = c = \frac{1}{\sqrt{2}}$ . The  $\nu_e - \nu_\mu$  level crossing takes place at  $\rho = 0$ . As before  $\nu_\tau - \nu_{2m}$  resonance is also at  $\rho \simeq 0$ . The Hamiltonian of  $\nu_\tau - \nu_{2m}$  system is simplified:  $\nu_{1m}$  decouples since at small densities  $\alpha \approx 0$ .

Another possibility,  $m_{e\tau} > m_{\mu\tau}$ , is realized in case of inverse hierarchy of couplings,  $f_{e\tau} > f_{\mu\tau}$ . Now  $\theta > \frac{\pi}{4}$  and  $\nu_e - \nu_\mu$  level crossing is at negative  $\rho$ ;  $e\mu$ -resonance conversion takes place in the antineutrino channel. Crossing of the  $\nu_\tau$ -  $\nu_{2m}$ - levels occurs at  $\rho \simeq 0$  as before. However these possibilities are disfavored by data from SN1987A [15].

## 4 Neutrino transitions in medium with variable density.

*Adiabatic conversion.* Mixing matrix and the eigenvalues obtained in sect.3 allow to get

the probabilities for oscillations in uniform medium as well as for the adiabatic conversion in medium with varying density. The probability of the adiabatic conversion  $P(\nu_i \rightarrow \nu_j)$  averaged over oscillations is determined by the mixing in the initial moment,  $S(\rho_0)$ , and in the final moment,  $S(\rho_f)$ :

$$P^{ad}(\nu_i \rightarrow \nu_j) = \sum_k |S_{jk}(\rho_f) S_{ik}(\rho_0)|^2. \quad (23)$$

Suppose  $\nu_e$  is produced at the density  $\rho_0$  and propagates adiabatically to  $\rho = 0$ , then substituting  $S(\rho_0) = S_{12}(\theta_m^0)$  and  $S(\rho_f) = S_0$  in (23) one finds the  $\nu_e \rightarrow \nu_e$  survival probability

$$P^{ad}(\nu_e \rightarrow \nu_e) = (1 - \frac{3}{2}s^2) \cos^2 \theta_m^0 + \frac{1}{2}s^2, \quad (24)$$

where  $\theta_m^0 \equiv \theta_m(\rho_0)$  and the mixing angle  $\theta_m$  is determined in (16). For  $\rho_0 \gg 1$  the mixing angle is  $\theta_m^0 \approx \frac{\pi}{2}$ , and from (24) one gets  $P \approx \frac{1}{2}s^2$ ; it differs by factor  $\frac{1}{2}$  from  $2\nu$  conversion probability due to maximal mixing oscillation (conversion) between two nearly degenerated states. When  $\rho \rightarrow 0$  ( $\theta_m \rightarrow \theta$ ), the probability (24) reduces to averaged vacuum probability (11). Note that at  $s^2 = \frac{2}{3}$  the probability does not depend on matter effects:  $P^{ad} = 1/3$ . At  $s^2 < \frac{2}{3}$  ( $s^2 > \frac{2}{3}$ ) the probability  $P^{ad}(\nu_e \rightarrow \nu_e)$  decreases (increases) with  $\rho_0$  increase.

The  $(\nu_e \rightarrow \nu_\mu)$ -transition probability,  $P^{ad}(\nu_e \rightarrow \nu_\mu)$ , can be found from (24) by the interchange  $s^2 \leftrightarrow c^2$ ; the interchange  $c_m \leftrightarrow s_m$  gives the probability of the transition  $P^{ad}(\nu_\mu \rightarrow \nu_e)$ . The antineutrino transitions are also described by (24); the mixing angle of antineutrinos in matter is smaller than that in vacuum, e.g. for  $\rho_0 \gg 1$  one has  $\theta_m \approx 0$  and  $P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx \cos^2 \theta$ .

*Adiabaticity violation.* Let us consider the general case taking into account the effects of adiabaticity violation and oscillations. The task is essentially simplified due to the existence of two different scales of  $\Delta m^2$ . As it was mentioned in sect.3 in a given density region only one mixing angle changes appreciably, whereas two others are “frozen”. Consequently, the three neutrino task is reduced to two neutrino tasks. In this case one can introduce partial adiabaticity parameter  $\kappa_{ij}$  that determine the probability of a jump between two given levels  $H_i, H_j$  as

$$\kappa(\rho)_{ij} = \frac{\Delta H_{ij}(\rho)}{\dot{\psi}_{ij}(\rho)} \cdot \frac{m_0^2}{2E}, \quad (25)$$

where  $\dot{\psi} \equiv \frac{d\psi}{dx}$  determines the change of the level mixing in a given density region and  $\Delta H_{ij}(\rho)$  is the level splitting.

In the region of large densities,  $\rho \gg 4\epsilon/\tan\theta$ , the change of mixing is stipulated mainly by  $\theta_m$ , i.e.  $\psi \equiv \theta_m$ , and in the  $e - \mu$  resonant point one gets using (16):

$$\kappa_{12}^R = \frac{2\sin^2 2\theta}{\dot{\rho}} \cdot \frac{m_0^2}{2E}. \quad (26)$$

As can be shown for a not too small mixing angle  $\theta$  the adiabaticity for the 2-3 levels is fulfilled much better than for the 1-2 levels so that with increasing  $\dot{\rho}$  the adiabaticity starts to be broken first for the 1 - 2 levels and then for the 2 - 3 levels. In the case of complete adiabaticity for all levels the neutrino state produced at  $\rho_0 \gg 1$  as  $\nu_e$  follows the  $H_2$  level (fig.1). If the adiabaticity of  $\nu_{2m}^0 - \nu_\tau$  system is broken then neutrino evolves along the  $H_2^0$  trajectory which is actually very close to  $H_2$  for  $\rho \gg \epsilon/s^2$ . Consequently, in this region it does not matter whether the 2 - 3 level adiabaticity is broken or not. In case of a strong adiabaticity violation for 1 - 2 levels in  $e\mu$ -crossing region the neutrino state follows the  $\nu_e$ -trajectory.

In the region of small densities,  $\rho \lesssim 4\epsilon/\tan\theta$ , one has  $\psi = \beta$ . Here  $\theta_m \approx \theta$ ,  $\alpha \approx 0$ . The adiabaticity parameter for  $\nu_{2m}^0 - \nu_\tau$  levels in resonance ( $\rho \approx 0$ ):

$$\kappa_{23}^R \approx \frac{32\epsilon^2 \cos^2 \theta}{\dot{\rho}} \cdot \frac{m_0^2}{2E} \quad (27)$$

is much smaller than  $\kappa_{12}(\rho = 0)$ , i.e. the adiabaticity can be broken for  $\nu_{2m}^0 - \nu_\tau$  levels, whereas  $\nu_{1m}$  propagates adiabatically.

Another circumstance which simplifies the task is the maximal mixing of  $\nu_{2m}^0$  and  $\nu_\tau$  levels at  $\rho = 0$ . This ensures that the probabilities of the transitions with zero final density,  $\rho_f = 0$ , averaged over the oscillations do not depend on the adiabaticity condition in the  $\nu_{2m}^0 - \nu_\tau$  system (see Appendix).

*The probabilities of conversion.* Keeping in mind possible applications to the solar, supernova and the atmospheric neutrinos we will consider the propagation of the electron neutrino,  $\nu_e$ , produced at some density  $\rho_0$  towards zero density,  $\rho_f = 0$ . In general the adiabaticity may be broken in  $e\mu$ -resonance region as well as in the region of small densities ( $\rho \sim 0$ ). Leaving the medium the neutrinos will oscillate in vacuum, and moreover, the oscillations induced by small mass splitting may not be averaged out.

According to (15) the decomposition of the initial neutrino state over the instantaneous eigenstates is

$$\nu_0 = \nu_e \approx \cos \theta_m^0 \nu_{1m}^0 + \sin \theta_m^0 \nu_{2m}^0, \quad (28)$$

where  $\theta_m^0$  is the mixing angle in the production point (the admixture of  $\nu_{3m}$ , being of the order of  $\epsilon$ , is practically unessential). Let us introduce some density  $\rho'$  ( $\rho_b < \rho' \ll 1$ ) so that at  $\rho < \rho'$  the 1-2 level adiabaticity is restored or the change of the 1-2 mixing is negligibly small. As the result of propagation over the large density region one gets then at  $\rho'$  the state

$$\nu(\rho') = (A_{11}c_m^0 + A_{21}s_m^0)e^{i\phi'_m} \nu_{1m}^0(\rho') + (A_{12}c_m^0 + A_{22}s_m^0)\nu_{2m}^0(\rho'), \quad (29)$$

where  $A_{ij}$  ( $i,j = 1, 2$ ) are the amplitudes of transitions between the levels  $\nu_{1m}^0$  and  $\nu_{2m}^0$ ,  $\phi'_m$  is the phase,  $c_m^0 \equiv \cos \theta_m^0$  etc.. The amplitudes satisfy the relation:

$$|A_{12}|^2 = |A_{21}|^2 = 1 - |A_{22}|^2 \equiv P_{12}. \quad (30)$$

The jump probability  $P_{12}$  can be approximated by the Landau-Zener probability [19] (or its modifications):

$$P_{12} \approx P_{LZ} \equiv \exp(-\frac{\pi}{2}\kappa_{12}^R), \quad (31)$$

where  $\kappa_{12}^R$  is the adiabaticity parameter in resonance (26). For the adiabatic transitions one has  $A_{ij} = \delta_{ij}$ .

In the region of small densities,  $\rho \lesssim \rho'$ , the state  $\nu_{1m} \approx \nu_{1m}^0$  propagates adiabatically so that its admixture does not change, and at zero density one gets  $\nu_{1m} \approx \nu_1$ . The evolution of  $\nu_{2m}^0$  state is described by matrix (19). The result of its propagation to zero density can be presented as

$$\nu_{2m}^0 \rightarrow |a_{22}|e^{i\phi_m} \nu_2 + |a_{23}| \nu_3, \quad (32)$$

where  $a_{22}$  and  $a_{23}$  are the amplitudes of the transitions  $\nu_{2m} \rightarrow \nu_2$  and  $\nu_{2m} \rightarrow \nu_3$  correspondingly. In case of the adiabatic propagation  $a_{22} = 1$ ,  $a_{23} = 0$ . Note that neutrino crosses only half of the 2-3 resonance region and therefore the jump probability equals half of  $P_{LZ}$ :  $P_{23} \equiv |a_{23}|^2 \approx \frac{1}{2}P_{LZ}$ . Combining (32) and (29) one gets the neutrino state at the edge of medium:

$$\nu(0) = (A_{11}c_m^0 + A_{21}s_m^0)e^{i\phi'_m} \nu_1 + (A_{12}c_m^0 + A_{22}s_m^0)(|a_{22}|e^{i\phi_m} \nu_2 + |a_{23}| \nu_3). \quad (33)$$

Further propagation in vacuum results in changes of phases only<sup>2</sup>:  $\phi_m \rightarrow \phi = \phi_m + \phi_{vac}$  and  $\phi'_m \rightarrow \phi' = \phi'_m + \phi'_{vac}$ , where

$$\phi_{vac} = \frac{\Delta m_{23}^2}{2E} L \quad (34)$$

is phase difference acquired at a distance  $L$  in vacuum. Using (33) and the vacuum mixing matrix (7) one can obtain the probabilities of different transitions. In particular,  $(\nu_e \rightarrow \nu_e)$ -survival probability averaged over short length oscillations (phase  $\phi'$ ) is

$$P(\nu_e \rightarrow \nu_e) \equiv |<\nu_e|\nu(0)>|^2 = c^2 (\cos^2 \theta_m^0 - P_{12} \cos 2\theta_m^0) + \\ + \frac{1}{2}s^2 (\sin^2 \theta_m^0 + P_{12} \cos 2\theta_m^0) (1 + 2|a_{22}a_{23}| \cos \phi), \quad (35)$$

where we have taken into account (30) as well as similar normalization condition for  $a_{ij}$ . The probability averaged over  $\phi$ :

$$\bar{P}(\nu_e \rightarrow \nu_e) = \left(1 - \frac{3}{2}s^2\right) [\cos^2 \theta_m^0 - P_{12} \cos 2\theta_m^0] + \frac{s^2}{2} \quad (36)$$

does not depend on  $a_{ij}$  at all in accordance with the general statement proved in the Appendix. At  $P_{12} = 0$  (adiabatic propagation in the region of large densities) the result (36) coincides with (24). Using (36) we can rewrite the probability (35) as

$$P(\nu_e \rightarrow \nu_e) = \bar{P} + s^2 (\sin^2 \theta_m^0 + P_{12} \cos 2\theta_m^0) \sqrt{P_{23}(1 - P_{23})} \cos \phi, \quad (37)$$

where  $P_{23} \equiv |a_{23}|^2$ . If the adiabaticity for  $\nu_{2m}^0 - \nu_\tau$  levels is strongly broken then one has  $a_{22} = a_{23} = \frac{1}{\sqrt{2}}$  or  $P_{23} = \frac{1}{2}$  which corresponds to  $\nu_{2m}^0 \rightarrow \nu_2^0 = \frac{\nu_2 + \nu_3}{\sqrt{2}}$  transition in medium. Substituting  $P_{23} = \frac{1}{2}$  in (37) one gets

$$P(\nu_e \rightarrow \nu_e) = \bar{P} + \frac{1}{2}s^2 (\sin^2 \theta_m^0 + P_{12} \cos 2\theta_m^0) \cos \phi. \quad (38)$$

In this case the depth of vacuum oscillations is maximal.

*Energy dependence of the suppression factors.* Let us consider the dependence of the probability (38) on the neutrino energy (fig.2).  $P(E)$  is the oscillating curve inscribed into the band between  $P^{max}$  and  $P^{min}$  (the oscillations are stipulated by change of  $\phi$ ). For  $\cos(\frac{\Delta m^2}{2E}L) = 1$  one gets from (38)

$$P^{max} = \frac{1}{2} + \left(\frac{1}{2} - P_{12}\right) \cos 2\theta_m^0 \cos 2\theta \quad (39)$$

---

<sup>2</sup>The lost of coherence due to wave packet spread is reduced to the averaging effect.

which coincides with  $2\nu$ -probability  $P_{2\nu}$ , i.e.  $P_{2\nu}$  gives the upper bound for  $3\nu$ -survival probability:  $P_{3\nu} < P_{2\nu}$ . The width of the band,

$$\Delta P \equiv P^{max} - P^{min} = s^2(\sin^2 \theta_m^0 + P_{12} \cos 2\theta_m^0), \quad (40)$$

is proportional to  $\sin^2 \theta$ , and consequently, with diminishing  $\theta$  the  $3\nu$ -probability converges to  $2\nu$ -probability. For neutrinos propagating in matter with monotonously changing density  $P^{max} = P_{2\nu}(E)$  has the form of pit (fig.2) [14]. Outside the pit the probability approaches 1 on the right hand side and the vacuum value,  $\sin^2 2\theta/2$ , on the left hand side. The position of the left (adiabatic) edge of the pit,  $E_{ad}$ , is fixed via the resonant condition by the density in the production point. The position of the right (nonadiabatic) edge  $E_{na}$  is determined by the adiabaticity condition. As follows from (40),  $\Delta P = s^2 \sin^2 \theta_m^0$  in the adiabatic region. In particular, for small energies outside the pit, where the matter effect is weak, one has  $\Delta P = s^4$ . Maximal width of the strip,  $\Delta P = s^2$ , is at the bottom of the pit when  $\rho_0 \gg \rho_R$ , therefore at the bottom:  $P^{min} = 0$ . In the nonadiabatic region  $\Delta P$  decreases with enhancement of the adiabaticity violation:  $\Delta P \simeq s^2(1 - P_{12})$ .

The position of the first (broadest) minimum of the oscillating curve,  $E_m$ , is determined from the condition  $\frac{l_\nu}{2} = L$ , where  $l_\nu$  is the oscillation length in vacuum. Explicitly one has  $E_m = \frac{\Delta m_{23}^2 L}{2\pi}$ ; the first maximum is at  $E_m/2$  etc.. Mutual position of the pit and the modulating curve is fixed by the ratio of energies

$$\frac{E_{ad}}{E_m} = \frac{2\pi \cos 2\theta}{L\sqrt{2}G_F n_0} \frac{\Delta m_{12}^2}{\Delta m_{23}^2} \approx \frac{\Delta m_{12}^2 l_0}{\Delta m_{23}^2 L}, \quad (41)$$

where  $l_0 \equiv 2\pi/\sqrt{2}G_F n_0$  is the refraction length in the neutrino production point. If  $E_m \gg E_{na} > E_{ad}$ , fast oscillations of the modulating curve are practically averaged in the energy region of the pit and  $P \approx \bar{P}$ . For  $E_m \sim E_{ad}$  one predicts the observable modulations of the pit. If  $E_m \ll E_a$  long length oscillations are not developed and  $P \rightarrow P_{2\nu}$ .

Similarly one can analyze the transitions of  $\nu_\mu$  and  $\nu_\tau$ . In particular,  $\nu_\tau$  is converted mainly in the region of small densities, and the probability of  $\nu_\tau \rightarrow \nu_e$  transitions averaged over large scale splitting equals

$$P(\nu_\tau \rightarrow \nu_e) = \frac{s^2}{2} \left[ 1 + \sqrt{P_{23}(1 - P_{23})} \cos \phi \right]. \quad (42)$$

For the antineutrino channel (negative  $\rho$ ) there is no  $\nu_e - \nu_\mu$  level crossing; one can consider adiabatic evolution of  $\bar{\nu}_{1m}$  and  $\bar{\nu}_{2m}^0$  states and as the result  $P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx P^{ad} = \cos^2 \theta$ .

If  $f_{e\tau} \sim f_{\mu\tau}$ , the  $e\mu$ -mixing becomes maximal; both resonant regions are at  $\rho = 0$ . The above consideration (reduction to two neutrino tasks) is valid due to difference in the widths of the resonance layers. Indeed in  $\nu_\tau - \nu_{2m}^0$  crossing region the change of the  $\nu_e - \nu_\mu$  mixing is negligibly small.

## 5 Applications

The results of the solar neutrino experiments can be reconciled with predictions of the Standard Solar Models in terms of the resonant flavor conversion  $\nu_e \rightarrow \nu_\mu(\nu_\tau)$  with parameters (two neutrino mixing):  $\Delta m^2 = (0.4 - 1.2) \cdot 10^{-5}\text{eV}^2$ ,  $\sin^2 2\theta = (0.1 - 1.5) \cdot 10^{-2}$  “small mixing solution” and  $\Delta m^2 = (0.5 - 3) \cdot 10^{-5}\text{eV}^2$ ,  $\sin^2 2\theta = (0.60 - 0.85)$  “large mixing solution” [20]. However taking into account the uncertainties of both the predictions and the experimental data, one should consider more wide region of the parameters. The deficit of  $\nu_\mu$  in the atmospheric neutrino flux testifies for oscillations  $\nu_\mu \rightarrow \nu_\tau$ , with  $\Delta m^2 = (0.5 - 3) \cdot 10^{-2}\text{eV}^2$ ,  $\sin^2 2\theta = (0.4 - 0.6)$  or for oscillations  $\nu_\mu \rightarrow \nu_e$  with  $\Delta m^2 = (0.5 - 3) \cdot 10^{-2}\text{eV}^2$ ,  $\sin^2 2\theta = (0.3 - 0.8)$  [21]. The formation of large scale structure of the Universe implies the existence of hot component of dark matter and the relic neutrinos with  $m \sim (2 - 7)$  eV can play such a role [22].

Let us confront these results with predictions of the Zee-model for different values of  $m_0$ . As follows from (2, 4), basic mass scale is  $m_0 = 7 \cdot 10^4 \cdot (I\sqrt{f_{e\tau}^2 + f_{\mu\tau}^2})$  eV, therefore depending on values of couplings  $f_{ij}$  as well as the parameters of the scalar sector one can get for  $m_0$  any value below  $\sim 10^4$  eV. For  $f_{e\mu} \leq f_{e\tau} \leq f_{\mu\tau}$  the ratio of the mass squared difference (8) may be in the range  $10^{-5} - 5 \cdot 10^{-3}$ , unless one introduces very strong hierarchy of couplings. Also under this condition the mixing angle is in the region  $\sin^2 2\theta = 10^{-3} - 1$ .

Three regions of  $m_0$  values are of special interest.

*Cosmologically interesting mass scale:*  $m_0 \sim (1 - 30)$  eV. The components  $\nu_2$  and  $\nu_3$  with masses  $m_0$ , can form the hot dark matter. Since  $\Delta m_{13}^2 \sim \Delta m_{12}^2 \sim m_0^2 \gtrsim 1\text{eV}^2$ , the mixing angle,  $\theta$ , is restricted by the accelerator oscillation experiments: e.g.  $\sin^2 2\theta < 2 \cdot 10^{-3}$  at  $m_0 = 3$  eV,  $\sin^2 2\theta < 6 \cdot 10^{-3}$  at  $m_0 = 1$  eV etc. [23]. For  $\sin^2 2\theta > 10^{-5}$  one expects strong resonant conversions  $\nu_e \rightarrow \nu_\mu$  and  $\nu_\mu \rightarrow \nu_e$  in the inner parts of the collapsing star. Such

a conversion results in permutation of the  $\nu_e$ - and  $\nu_\mu$ - energy spectra and therefore in the increase of average energy of the electron neutrinos. This will have two consequences: (i) the increase of the energy release due  $\nu_e - e$ - scattering which may help to expel the envelope, (ii) the formation of the proton-rich medium due to dominant  $\nu_e n \rightarrow ep$  scattering. The latter will forbid the r-processes responsible for nucleosynthesis of heavy elements [24]. If the inner part of collapsing stars is the only place of the r-processes, then the indicated conversion should be suppressed and one gets the bound on mixing angle  $\sin^2 2\theta < 10^{-4} - 10^{-5}$  [24]. The value of  $\Delta m_{23}^2$ , can be naturally in the region responsible for the atmospheric neutrino problem. The suppression of the muon neutrino flux due to  $\nu_\mu - \nu_\tau$ -oscillations is determined by the averaged vacuum probability (11) and taking into account the indicated bounds on  $\theta$  one gets:  $P \approx 0.5$  which is actually outside the region of the best fit of all the data. The suppression of the solar  $\nu_e$ -flux fixed by averaged vacuum probability (11) is very weak:  $P(\nu_e \rightarrow \nu_e) \geq 0.92$ .

*Atmospheric neutrino mass scale:*  $m_0 \simeq (0.3 - 1) \cdot 10^{-1}$ eV.  $m_0^2$  is in the region of the solution of the atmospheric neutrino problem  $\Delta m_{13}^2 \approx m_0^2 \simeq (10^{-3} - 10^{-2})$ eV $^2$ . The deficit of  $\nu_\mu$  can be explained by  $\nu_\mu \leftrightarrow \nu_e$  - oscillations, with  $\sin^2 2\theta = 0.3 - 0.8$ .

For  $\Delta m_{23}^2 = (10^{-6} - 10^{-4})$ eV $^2$  the suppression factor for solar neutrinos is determined by the adiabatic probability (24) which coincides at  $\theta_m \approx \theta$  with averaged vacuum probability (11). Solar neutrino spectrum is outside the pit at small energies. Although  $\Delta m_{23}^2$  is in the region of strong matter effect, the averaged probability  $P_\odot$ , practically does not depend on matter density and on the neutrino energy as well. For values of  $\theta$  needed to solve the atmospheric neutrino problem (at  $s^2 < c^2$ ) one gets from (11)  $P_\odot = 0.56 - 0.83$ . Taking into account an additional contribution to the  $\nu e$ -scattering from neutral currents one predicts the following suppression factors ( $R \equiv \text{data}/\text{SSM}$ ) for Ga-, Ar-, production rates and  $\nu e$ -signal:

$$R_{Ge} = R_{Ar} \simeq P_\odot \sim 0.56 - 0.83, \quad R_{\nu e} \simeq 0.62 - 0.86. \quad (43)$$

There is no distortion of energy spectrum of boron neutrinos. The predictions (43) fit rather well all the results except the one of Homestake experiment.

In case of very small splitting  $\Delta m_{23}^2 \ll 10^{-9}$  eV $^2$ , there is no averaging over long length oscillations and the probability is modulated with the amplitude  $s^4/2$  (see (40) and further discussion). For the indicated mixing angles the amplitude of modulations is  $\sim$

$(0.25 - 4.5) \cdot 10^{-2}$ . So, one may expect up to  $\sim 10\%$  distortion of boron neutrino spectrum and change of the relations (43). For example, at  $\sin^2 2\theta = 0.8$  and for certain values of  $\Delta m^2$  one may get  $R_{Ge} = 0.54$ ,  $R_{Ar} = 0.51$ , and  $R_{\nu_e} = 0.58$ . Further increase of  $\sin^2 2\theta$ , although enhances the amplitude of modulations, results in the stronger suppression of the atmospheric  $\nu_\mu$ -signal as well as the signal in gallium experiments.

As in the previous case one predicts strong resonant conversions  $\nu_e \rightarrow \nu_\mu$  and  $\nu_\mu \rightarrow \nu_e$  of the neutrinos from the collapsing cores of stars. The corresponding permutation factor which characterizes the interchange of  $\nu_e$ - and  $\nu_\mu$ - energy spectra [15] equals  $p = 0.75 - 0.90$ . Moreover, since the mixing is rather large one expects an appreciable permutation of the antineutrino spectra:  $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ ; the permutation factor  $\bar{p} = 0.08 - 0.25$  results in even better description of data from SN1987A [27], [15].

Note that for indicated values of  $m_0$  the conversions take place now in more external layers, so that there is no problem with r-processes.

Another value of the mixing angle which follows from explanation of the atmospheric neutrino data ( $s^2 > c^2$ , this corresponds to the consideration in [25]) gives  $P_\odot = 0.17 - 0.44$  and  $P_{\nu_e} = 0.29 - 0.51$ . The result contradicts to the observed gallium production rate. Moreover the  $e - \mu$ -resonance is in the antineutrino channel which results in strong permutation ( $\bar{p} = 0.6 - 0.8$ ) of the  $\nu_e$ -,  $\nu_\mu$ - energy spectra. The SN1987A data give the bound  $\bar{p} < 0.4$  [15].

*Solar neutrinos mass scale:*  $m_0 \simeq (10^{-2} \sim 10^{-3})$  eV. the largest mass splitting,  $\Delta m_{13}^2 \sim (10^{-6} - 10^{-4})$  eV $^2$ , is in the region of the resonant effect inside the Sun. If  $\Delta m_{23}^2 \gg 10^{-10}$  eV $^2$ , averaging over long length vacuum oscillations takes place and the suppression factor for  $\nu_e$ -flux is determined by the probability  $\bar{P}$  (36). It can be rewritten as:

$$P = P_{2\nu} - \frac{s^2}{2} [\sin^2 \theta_m + P_{12} \cos 2\theta_m], \quad (44)$$

where  $P_{2\nu}$  is the suppression factor for two neutrino mixing. One can easily construct the  $3\nu$ -suppression pit using (44) and the results for two neutrino mixing. For small  $\theta$  the effect of the third neutrino is negligibly small and  $P \simeq P_{2\nu}$ . The Zee-mass matrix reproduces small mixing solution of the solar neutrino problem for two neutrinos. The deviation from the  $2\nu$  case is of the order of  $10^{-3}$  and to distinguish the Zee mechanism from the other mechanisms one can take into account the following facts. In the considered case there

is no oscillation solution of the atmospheric neutrino problem as well as no appreciable contribution of neutrinos to hot dark matter. Moreover, there is no manifestation of the third neutrino in the experiments with supernova neutrinos.

Large mixing solution is absent unless one admits large original flux of boron neutrinos. Indeed, since at the bottom of the pit  $P = \sin^2 \theta/2$  one needs two times bigger value of  $\sin^2 \theta$  in comparison with  $2\nu$  case to get the same suppression of Kamiokande signal, e.g. instead of  $\sin^2 2\theta = 0.7$  one should take  $\sin^2 2\theta = 0.994$ . However in this case the fluxes of the low energy neutrinos are strongly suppressed: in the pp-neutrino region one gets  $P \approx 0.375$ .

The situation is essentially different if  $\Delta m_{23}^2 \leq 10^{-9}\text{eV}^2$ , and consequently there is no averaging over the long-length oscillation at least in some part of the suppression pit (fig.2). One of the most interesting configurations is shown in fig.3 which corresponds to  $\Delta m_{23}^2/\Delta m_{12}^2 \sim 10^{-5}$  and  $\sin^2 2\theta = 0.8 - 0.9$ . First minimum of oscillating curve is at adiabatic edge, first maximum is outside the pit. If the Be-neutrino line is in first minimum of oscillating curve then the boron neutrinos are at the bottom of suppression pit outside the minimum, and the detected part of the pp- neutrino spectrum is in the first maximum. The signature of such a scenario is the strong suppression of the Be-neutrino flux, and the absence of the *distortion* of the high energy part of the boron neutrino spectrum in contrast with small mixing solution for two neutrinos. For pp-neutrinos one can get the suppression 0.55 - 0.60, so that total Ge-production rate could be about 50 - 70 SNU in agreement with present data.

Another possibility corresponds to the modulating curve shifted to larger energies, so that boron neutrinos are in the first maximum (pp-neutrinos are in averaging region) and the beryllium line is in the fastly oscillating region. If the oscillating curve is shifted to lower energies then one expect the distortion of the pp-neutrino spectrum.

Obviously in this case there is no solution of the atmospheric neutrino problem. For the indicated values of parameters one may expect complete or partial conversion of the supernova neutrinos  $\nu_e \leftrightarrow \nu_\mu$  depending on density profile of star.

The loop diagrams, similar to those generating the neutrino masses, will generate also the transition magnetic moments of the neutrinos. These moments are however restricted by  $\mu < 3 \cdot 10^{-15} \mu_B (m_0/1\text{eV})$ , where  $\mu_B$  is Bohr magneton [26], so that even for  $m_0 \approx 10 \text{ eV}$  the effects of spin-flip on the solar and atmospheric neutrinos are negligibly small.

*Implications to the parameters of the Zee model.* Let us comment on possible implications of the above results to the original Zee-model. The expression for  $m_0$  can be rewritten as

$$f_{\mu\tau} \sim 1.3 \cdot 10^{-5} \left( \frac{m_0}{1\text{eV}} \right) \cdot \frac{1}{I} . \quad (45)$$

Moreover, the dimensionless parameter  $I$  can be of the order 1 when all mass parameters entering  $I$  are at the electroweak scale and  $v_u \sim v_d$ . According to (45) a value  $I = 1$  gives the lower bound on  $f_{\mu\tau}$ . In principle,  $f_{\alpha\beta}$  can be as large as 1;  $f_{\mu\tau} \approx 1$  gives the lower bound on  $I$ , and consequently, the upper bound on the mass of charged scalar  $M_2$ . Let us estimate the range of the parameters.

For  $m_0$  in cosmologically interesting domain one has  $f_{\mu\tau} > (0.13 - 4) \cdot 10^{-4}$  and  $M_2 < (2 - 8) \cdot 10^4$  GeV. The indicated values of mass squared differences and accelerator bounds on  $\theta$  correspond to  $f_{e\mu} \sim f_{e\tau} \sim (0.03 - 0.1)f_{\mu\tau}$ . If  $m_0$  is in the region of the atmospheric neutrino problem one gets  $f_{\mu\tau} > (0.4 - 1.3) \cdot 10^{-6}$  and  $M_2 < (0.8 - 2.4) \cdot 10^5$  GeV. All constants can be of the same order:  $f_{\mu\tau} \sim f_{e\tau} \sim f_{e\mu}$ . Alternatively, if there is no averaging over long length oscillations  $f_{e\mu} = 10^{-4}f_{\mu\tau}$ . For  $m_0$  in the region of the solutions of the solar neutrino problem the parameters are  $f_{\mu\tau} > (0.13 - 1.3) \cdot 10^{-8}$  and  $M_2 < (3 - 4) \cdot 10^6$  GeV. Small mixing solution implies  $f_{e\tau} < 0.1f_{\mu\tau}$ . Large mixing solution with modulations by long length oscillations is realized when  $f_{e\tau} \sim f_{\mu\tau}$  and  $f_{e\mu} \sim 10^{-2}f_{\mu\tau}$ .

## 6 Discussion and Conclusions

1. Zee-matrix in matter (for  $\epsilon \ll 1$ ) is an example of “solvable”  $3\nu$ -task. This allows to trace some interesting features of the dynamic of propagation, in particular, the effect of strong influence of one resonance on another. Dominant  $\nu_e - \nu_\mu$  mixing shifts the resonance for  $\nu_\tau$  to zero density, thus changing a naive picture of level crossings.
2. In the supersymmetric generalization of the model new diagrams appear with sleptons and higgsino in the loops. General structure of mass matrix is the same as (1). Note that Zee singlet can be embedded in the SU(5) GUT scheme by introducing the antisymmetric 10-plet of scalars.
3. Zee-matrix can be considered as an alternative to the one generated by the see-saw mechanism. Let us note for a sake of completeness that in principle the matrix (1) can be

reproduced by the see-saw mechanism too. For this one needs a special structure of the Majorana mass matrix of the right components,  $M_R$ . Namely, in the Dirac neutrino basis (where the Dirac matrix is diagonal)  $M_R$  should have zero determinants of three submatrices:  $M_{ii}M_{jj} - M_{ij}^2 = 0$ , ( $i,j = 1,2,3$ ).

4. The mass matrices generated at the one loop level in the model with explicit R-parity violation [5] also differ from (1). The generic property of these matrices is the existence of nonzero diagonal elements which are not suppressed in comparison with nondiagonal elements. If purely lepton couplings dominate over couplings of quark and lepton supermultiplets, the elements  $m_{e\tau}$ ,  $m_{\mu\tau}$ , are naturally suppressed with respect to others by factor of  $m_\mu/m_\tau$ . Moreover, the element  $m_{\tau\tau}$  has even stronger suppression:  $(m_\mu/m_\tau)^2$ . Such a matrix allows to explain both solar and the atmospheric neutrino deficits [28] according to the scenario suggested in [25].

Similar structure of mass matrix appears in the model with two loops generation of the neutrino masses [8].

5. Practically all the extensions of the standard model imply the existence of neutral fermions which can play the role of right-handed neutrinos. In this case the see-saw mechanism is obtained and in addition to the radiative mass terms one gets the see-saw contributions to the neutrino mass matrix:  $m_{ss} \approx m_D M^{-1} m_D^T$ . The biggest term is  $m_{\tau\tau} \approx \frac{m_{3D}^2}{M}$ , where the Dirac mass,  $m_{3D}$ , can be as large as the top quark mass. Depending on values of parameters which may be in the range  $m_D = (1 - 10^2)$  GeV and  $M = (10^{10} - 10^{18})$  GeV one can get negligibly small, or comparable with  $\epsilon m_0$ , or even dominant see-saw contribution. For example, at  $m_{3D} \sim 100$  GeV and  $M = 10^{18}$  GeV one has  $m_{\tau\tau} \sim 10^{-5}$  eV which is of the order of  $\epsilon m_0$  for  $m_0 < 10^{-3}$  eV. If  $M < 10^{16}$  GeV this contribution becomes dominant.

Let us comment on the simplest possibility when only one additional element,  $m_{\tau\tau}$ , is important. Such a situation is realized if the Majorana mass matrix of the RH-components has no strong hierarchy. In this case one can get easily both large  $\mu - \tau$  - mixing angle,  $\theta_{\mu\tau}$ , needed to solve the atmospheric neutrino problem and small mixing solution of the solar neutrino problem. Indeed, now  $\mu\tau$ -mixing is of the order of  $m_0/m_{ss}$  and large value of  $\theta_{\mu\tau}$  follows from the fact that the see-saw and the radiative contributions are of the same order. Moreover, for small  $e\mu$  mixing the following relation between the ratio of masses and the

$\mu\tau$ - mixing exists:

$$\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \sim \frac{m_2^2}{m_3^2} \sim \left[ \frac{1 - \cos 2\theta_{\mu\tau}}{1 + \cos 2\theta_{\mu\tau}} \right]^2. \quad (46)$$

For  $\sin^2 2\theta_{\mu\tau} > 0.4$  one gets  $\frac{m_2^2}{m_3^2} > 0.01$  which is roughly consistent with desirable value. The ratio can be further corrected if one takes into account the see-saw contributions to other elements of matrix.

*In conclusion*, we have considered the properties of propagation of the neutrinos with Zee-like mass matrix. Crucial features of the matrix are zero (small) diagonal elements and natural hierarchy of nondiagonal elements. It can be considered as an alternative to the matrix generated by the see-saw mechanism as well to other radiative mechanisms. The probabilities of the conversions induced by the Zee-mass matrix as the functions of the neutrino energies can be represented as two neutrino probabilities modulated by the oscillating curve related to the long length oscillations.

The Zee-mass matrix does not allow to explain all three problems related to the solar atmospheric and relic neutrinos simultaneously. If two heavy components are in the cosmologically interesting domain then one can get for atmospheric muon neutrinos the suppression 1/2 with no appreciable effect for solar neutrinos. The Zee-mass matrix allows to fit well the atmospheric neutrino data if the masses of heavy components are in the region  $\sim 0.1$  eV. In this case one predicts energy independent suppression of  $\nu_e$  flux at the level 0.6. For smaller  $m_0$  the matrix can reproduce with high precision small  $2\nu$ -mixing solution of the solar neutrino problem without any appreciable manifestations of the third neutrino. For large mixing  $\theta$  new configurations of the suppression appear. In particular, one may have strong suppression of the beryllium line and energy independent suppression of the high energy part of boron neutrino spectrum. In general, the modulations of the smooth energy dependence of the probabilities for  $2\nu$ -case are expected.

These features may allow to identify the Zee-mass matrix and consequently the corresponding mechanism of neutrino mass generation.

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## Appendix

Suppose the neutrino propagates in medium with density decreasing from  $\rho_i$  in the initial point to zero. Let us show that the averaged oscillation probability does not depend on the adiabaticity condition for the levels having maximal mixing in vacuum.

Indeed, the initial neutrino state can be written as

$$\nu_i = a\nu_{1m} + b\nu_{2m} + c\nu_{3m}.$$

Suppose  $\nu_{1m}$ - state propagates adiabatically, whereas the adiabaticity for  $\nu_{2m} - \nu_{3m}$  may be broken. The latter means that there are the transitions  $\nu_{2m} \leftrightarrow \nu_{3m}$  while the neutrino propagates to zero density. At zero density the neutrino state will have the form

$$\nu_f = a\nu_1 + b'\nu_2 + c'\nu_3.$$

In general  $|b'|$  and  $|c'|$  differ from  $|b|$  and  $|c|$ , however the normalization condition implies that

$$|b|^2 + |c|^2 = |b'|^2 + |c'|^2 = 1 - |a|^2.$$

The probability to find the neutrino  $\nu_\alpha \equiv a_\alpha \nu_1 + b_\alpha \nu_2 + c_\alpha \nu_3$  in final state averaged over the oscillations equals

$$P = |a_\alpha^\dagger a|^2 + |b_\alpha^\dagger b'|^2 + |c_\alpha^\dagger c'|^2.$$

Maximal mixing of  $\nu_2$  and  $\nu_3$  in  $\nu_f$  means that  $b_\alpha = c_\alpha$ , and then taking into account the normalization condition one gets

$$P = |a_\alpha^\dagger a|^2 + |b_\alpha|^2(|b|^2 + |c|^2) = |a_\alpha^\dagger a|^2 + |b_\alpha|^2(1 - |a|^2)$$

which does not depend on changes of  $b$  and  $c$ .

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## Figure Caption

Fig.1 Energy levels of the neutrinos with Zee-mass matrix as the functions of the matter density  $\rho$  (full lines). Dashed lines correspond to the levels in zero approximation over  $\epsilon$ . Dotted lines show the flavor levels. a). Small mixing angle  $\theta$ , b). for  $\theta = 45^\circ$ .

Fig.2 Survival probability  $P(\nu_e \rightarrow \nu_e)$  as the function of the neutrino energy for different values of mixing angle  $\theta$  (solid lines) a).  $\sin^2 2\theta = 0.64$ , b). $\sin^2 2\theta = 0.84$ . Dashed lines correspond to  $P^{max}$  and  $P^{min}$ , the averaged probability is shown by dotted line.

Fig.3 The suppression factor for large mixing and nonaveraged vacuum oscillations. Also shown is the solar neutrino spectrum (hatched).

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